1 Concepts of Motion

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time interval spent travelling}} = \frac{\delta}{\Delta t}$$

$\Delta\vec{r} = \vec{r}_\text{f} - \vec{r}_\text{i}$

$\langle\vec{v}\rangle = \frac{\Delta\vec{s}}{\Delta t} \qquad \langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t}$

2 Kinematics in One Dimension

For motion in a straight line:

$v_s = \frac{\mathrm{d}s}{\mathrm{d}t}$ =slope of position-time graph

$a_s = \frac{\mathrm{d}v_s}{\mathrm{d}t}$ =slope of velocity-time graph

$s_\text{f} = s_\text{i} + \int_{t_\text{i}}^{t_\text{f}} v_s\,dt = s_\text{i} +$ area under velocity curve from t_i to t_f

$v_{s,\text{f}} = v_{s,\text{i}} + \int_{t_\text{i}}^{t_\text{f}} a_s\,dt = v_{s,\text{i}} =$ area under accel. curve from t_i to t_f

Uniform motion:

$s_\text{f} = s_\text{i} + v_s\Delta t$

Uniformly accelerated motion:

$v_{s,\text{f}} = v_{s,\text{i}} + a_s\Delta t$

$s_\text{f} = s_\text{i} + v_{s,\text{i}}\Delta t + 0.5a_s(\Delta t)^2$

$v_{s,\text{f}}^2 = v_{s,\text{i}}^2 + 2a_s\Delta s$

Free fall: $a_y = -g = -9.80$ m/s^2

Motion on an inclined plane: $a_s = \pm g\sin\theta$

3 Vectors and Coordinate Systems

$\vec{A}_x = A_x\hat{\imath}$

$\vec{A}_y = A_y\hat{\jmath}$

$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{\jmath}$

$A_x = |\vec{A}|\cos\theta \qquad A_y = |\vec{A}|\sin\theta$

$A = \sqrt{A_x^2 + A_y^2} \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

4 Kinematics in Two Dimensions

$\vec{r} = x\hat{\imath} + y\hat{\jmath}$

$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}\hat{\imath} + \frac{\mathrm{d}y}{\mathrm{d}t}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$

$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}v_x}{\mathrm{d}t}\hat{\imath} + \frac{\mathrm{d}v_y}{\mathrm{d}t}\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$

Constant acceleration:

$x_\text{f} = x_\text{i} + v_{x,\text{i}}\Delta t + 0.5a_x(\Delta t)^2 \qquad y_\text{f} = y_\text{i} + v_{y,\text{i}}\Delta t + 0.5a_y(\Delta t)^2$

$v_{x,\text{f}} = v_{x,\text{i}} + a_x\Delta t \qquad v_{y,\text{f}} = v_{y,\text{i}} + a_y\Delta t$

Projectile motion:

$x_\text{f} = x_\text{i} + v_{x,\text{i}}\Delta t \qquad y_\text{f} = y_\text{i} + v_{y,\text{i}}\Delta t - 0.5g(\Delta t)^2$

$v_{x,\text{f}} = v_{y,\text{f}} = \text{constant} \qquad v_{y,\text{f}} = v_{y,\text{i}} - g\Delta t$

$v_{y,\text{f}}^2 = v_{y,\text{i}}^2 - 2g(y_\text{f} - y_\text{i})$

Relative motion:

$\vec{r}_\text{CB} = \vec{r}_\text{CA} + \vec{r}_\text{AB}$

$\vec{v}_\text{CB} = \vec{v}_\text{CA} + \vec{v}_\text{AB}$

$\vec{v}_\text{AB} = -\vec{v}_\text{BA}$

Circular motion:

$\theta = \frac{\ell_a}{r} \qquad \langle\omega\rangle = \frac{\Delta\theta}{\Delta t} \qquad \omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$

$\vec{v}_\text{t} = \omega r \qquad \vec{\alpha} = \frac{\mathrm{d}\omega}{\mathrm{d}t}$

Uniform circular motion (constant ω):

$\vec{v}_\text{t} = \frac{2\pi r}{T} \qquad \omega = \frac{2\pi\,[\text{rad}]}{T}$

$\theta_\text{f} = \theta_\text{i} + \omega\Delta t$

$\vec{a}_r = \frac{\vec{v}_\text{t}^2}{r} = \omega^2 r$

Non-uniform circular motion (constant $\vec{a}_\text{t}$):

$\vec{a}_r = \frac{\mathrm{d}\vec{v}_\text{t}}{\mathrm{d}t} = \vec{\alpha}r$

$\theta_\text{f} = \theta_\text{i} + \omega_\text{i}\Delta t + 0.5\vec{\alpha}(\Delta t)^2$

$\omega_\text{f} = \omega_\text{i} + \vec{\alpha}\Delta t$

$\omega_\text{f}^2 = \omega_\text{i}^2 + 2\vec{\alpha}\Delta\theta$

5 Dynamics I: Motion Along a Line

$\vec{F}_\text{net} = \sum_i \vec{F}_i = m\vec{a}$

$\vec{F}_g = mg$ (downward)

$W = mg\left(1 + \frac{a_y}{g}\right)$ (object with vertical acceleration)

A model for friction:

Static:	$\hat{F}_\text{s} = 0$ to $\mu_\text{s}\vec{F}_n$	(direction prevents motion)
Kinetic:	$\hat{F}_\text{k} = \mu_\text{k}\vec{F}_n$	(direction opposite motion)
Rolling:	$\hat{F}_\text{r} = \mu_\text{r}\vec{F}_n$	(direction opposite motion)
Drag:	$\vec{F}_\text{d} = \frac{\langle\vec{v}\rangle^2\rho A_\sigma C_\text{d}}{2}$	(direction opposite motion)

6 Newton's Third Law

$\vec{F}_\text{A on B} = -\vec{F}_\text{B on A}$

7 Dynamics II: Motion in a Plane

$F_{x,\text{net}} = \sum F_x = ma_x$

$F_{y,\text{net}} = \sum F_y = ma_y$

$F_{r,\text{net}} = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r$

$F_{t,\text{net}} = \sum F_t = ma_t$ (non-uniform) or 0 (uniform)

$F_{z,\text{net}} = \sum F_z = 0$

8 Impulse and Momentum

$\vec{P} = m\vec{v}$

$\vec{F} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t}$

$\vec{J}_x = \int_{t_\text{i}}^{t_\text{f}} F_x(t)\,dt =$ area under force curve $= \langle F\rangle\Delta t$

$\Delta\vec{P}_x = \vec{J}_x$ (impulse-momentum theorem)

$\vec{P} = \sum_i \vec{P}_i$

$\vec{P}_\text{f} = \vec{P}_\text{i}$ (conservation of momentum)

9 Energy

$E_\text{k} = 0.5m\vec{v}^2$

$E_\text{p} = mgh$

$E_\text{mech} = E_\text{k} + E_\text{p}$

$E_\text{k,f} + E_\text{p,f} = E_\text{k,i} + E_\text{p,i}$ (conservation of mechanical energy)

$F_{\text{sp},s} = -k\Delta s \qquad E_\text{p,sp} = 0.5k(\Delta s)^2 \qquad \Delta s = s - s_e$

Perfectly elastic 1D collisions (m_2 initially at rest):

$v_{x,\text{f},1} = \frac{m_1 - m_2}{m_1 + m_2}v_{x,\text{i},1} \qquad v_{x,\text{f},2} = \frac{2m_1}{m_1 + m_2}v_{x,\text{i},2}$

10 Work

$W = \int_{t_\text{i}}^{t_\text{f}} F_s\,ds =$ area under force-position curve

$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta$ (if $\vec{F}$ is a constant force)

$\Delta E_\text{k} = W_\text{net} = W_\text{cons} + W_\text{diss} + W_\text{ext}$ (work-kinetic energy theorem)

$\Delta E_\text{p} = E_\text{p,f} - E_\text{p,i} = -W_\text{cons}$ (i $\rightarrow$ f)

$F_s = \frac{\mathrm{d}E_\text{p}}{\mathrm{d}s}$

$Q = -W_\text{diss} = \hat{F}_\text{k}\Delta s$

$E_\text{k,f} + E_\text{p,f} + \Delta Q = E_\text{k,i} + E_\text{p,i} + W_\text{ext}$

$\text{P} = \frac{\mathrm{d}E_\text{sys}}{\mathrm{d}t} \qquad \text{P} = \frac{\mathrm{d}W}{\mathrm{d}t} = \vec{F}\cdot\vec{v} = |\vec{F}||\vec{v}|\cos\theta$

11 Newton's Theory of Gravity

$m' > m$

$F_{m' \text{ on } m} = F_{m \text{ on } m'} = \frac{Gm'm}{r^2}$

$g_\text{surface} = \frac{Gm'}{R^2}$

Circular orbit: $\vec{v} = \sqrt{\frac{Gm'}{r}} \qquad T^2 = \left(\frac{4\pi^2}{Gm'}\right)r^3$

12 Physical Constants

$g = 9.80$ m/s^2

$G = 6.67\times10^{-11}$ N $\cdot$ m$^2\cdot$ kg^{-2}

$m_{♁} = 5.98\times10^{24}$ kg

$R_{♁} = 6.37\times10^{6}$ m

13 Electric Charges

$\vec{F}_{q1q2} = \frac{k_e|q_1||q_2|}{r^2}\hat{e}_r$
$q = (N^+ - N^-)e$
$\vec{E} = \frac{\vec{F}}{q}$
$\vec{E} = k_e \frac{q}{r^2}\hat{e}_r$

14 The Electric Field

$\vec{E} = \sum_i \vec{E}_i$
$\vec{p} = q\vec{s} \quad \vec{s}: -q \longrightarrow +q$
Field on axis: $\vec{E} = k_e \frac{2\vec{p}}{r^3}$
Field in bisecting plane: $\vec{E} = k_e \frac{\vec{p}}{r^3}$
$q_\lambda = \frac{Q}{\ell} \quad q = \int q_\lambda \, d\ell$
$q_\sigma = \frac{Q}{A} \quad q = \iint q_\sigma \, dS$
$q_\rho = \frac{Q}{V} \quad q = \iiint q_\rho \, dV$
Uniform infinite line of charge:
$\vec{E} = k_e \frac{2q_\lambda}{r} \perp$ line
Uniform infinite plane of charge:
$\vec{E} = \frac{2q_\sigma}{2\varepsilon_0} \perp$ plane
Uniform sphere of charge:
$\vec{E} = k_e \frac{Q}{r^2}\hat{e}_r \quad r \geq R$
Parallel plate capacitor:
$\vec{E} = \frac{q_\sigma}{\varepsilon_0} \quad +q \longrightarrow -q$
Ring: $\vec{E}_z = k_e \frac{Qz}{(z^2+R^2)^{1.5}}$
Disk: $\vec{E} = \frac{q_\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$
$\vec{a} = \frac{q\vec{E}}{m}$
$\vec{\tau} = \vec{p} \times \vec{E} \quad |\vec{\tau}| = |\vec{p}||\vec{E}|\sin\theta$

15 Gauss's Law

$\Phi_E = \vec{E}\cdot\vec{A} = |\vec{E}||\vec{A}|\cos\theta$
$\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$
$\vec{E}$ at surface of a charged conductor: $\vec{E} = \frac{q_\sigma}{\varepsilon_0}$

16 The Electric Potential

Parallel plate capacitor:
$U_E = U_{E,0} + qEs \quad V = Es \quad E = \frac{\Delta V_{\text{plate}}}{\delta}$
$U_E = k_e \frac{q_1 q_2}{r} \qquad U_E = k_e \sum_{i<j} \frac{q_i q_j}{r_{ij}}$
$U_{\text{dipole}} = -\vec{p}\cdot\vec{E} = -|\vec{p}||\vec{E}|\cos\theta$
$V \equiv \frac{U_{q+\text{sources}}}{q}$
Point charge:
$V = k_e \frac{q}{r} \qquad V = k_e \sum_i \frac{q_i}{r_i}$

17 Potential and Field

$\Delta V = V(s_2) - V(s_1) = -\int_{s_1}^{s_2} E_s \, ds = -\int_1^2 \vec{E}\cdot d\vec{s}$
$E_s = -\frac{\mathrm{d}V}{\mathrm{d}t}$
$\Delta V = \frac{W_{\text{chemical}}}{q} = \mathcal{E}$ (ideal battery)
$C \equiv \frac{Q}{\Delta V_C} \qquad C = \varepsilon_R C_0$
Parallel plate capacitor: $C = \frac{\varepsilon_0 A}{\delta}$
$C_{\text{eq}} = \sum_i C_i$ (parallel)
$C_{\text{eq}} = \left(\sum_i \frac{1}{C_i}\right)^{-1}$ (series)
$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2 \qquad U_E = \frac{1}{2}E^2\varepsilon_0$

18 Current and Capacity

Electron Current:
$N_e = I_e \Delta t$
$I_e = \rho_{N,e} A_\sigma \vec{v}_d$
$\vec{v}_d = \frac{e\tau}{m_e}\vec{E}$
Conventional current:
$J = \frac{I}{A}$
$J = \rho_{N,e} e \vec{v}_d = \sigma E$
$\sigma = \frac{\rho_{N,e} e^2 \tau}{m_e} = \frac{1}{\rho}$
$E_{\text{wire}} = \frac{V_{\text{wire}}}{\ell}$
$I = \frac{\Delta V_{\text{wire}}}{R}$ where $R = \frac{\rho\ell}{A} = \frac{E}{J}$

19 Fundamentals of Circuits

$I = \frac{\Delta V}{R}$
Junction law: $\sum I_{\text{in}} = \sum I_{\text{out}}$
Loop law: $\Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0$
$P_{\text{battery}} = I\mathcal{E}$
$P_R = I\Delta V_R = I^2 R = \frac{(\Delta V)^2}{R}$
$R_{\text{eq}} = \sum_i R_i$ (series)
$R_{\text{eq}} = \left(\sum_i \frac{1}{R_i}\right)^{-1}$ (parallel)
$Q = Q_0 e^{-t/\tau} \quad I = I_0 e^{-t/\tau} \quad \tau = RC$

20 The Magnetic Field

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \left(\frac{\mu_0|q|v\sin\theta}{4\pi r^2}, \text{RHR}\right)$
$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2} = \left(\frac{\mu_0|I|(\Delta s)\sin\theta}{4\pi r^2}, \text{RHR}\right)$
$|\vec{B}|_{\text{straight wire}} = \frac{\mu_0}{2\pi}\frac{I}{r} \quad |\vec{B}|_{\text{coil centre}} = \frac{\mu_0}{2}\frac{N_t I}{R}$
$\vec{\mu} = AI \quad S \longrightarrow N$
$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$ on axis of dipole
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}}$
$|\vec{B}|_{\text{solenoid}} = \frac{\mu_0 N_t I}{\ell}$
$\vec{F}_{\text{on q}} = q\vec{v}\times\vec{B} = (|q|vB\sin\theta, \text{RHR})$
Cyclotron:
$f = \frac{q|\vec{B}|}{2\pi m} \quad r = \frac{mv}{q|\vec{B}|}$
$\vec{F}_{\text{wire}} = I\vec{\ell}\times\vec{B} = (I\ell B\sin\theta, \text{RHR})$
$|\vec{F}|_{\text{parallel wires}} = \frac{\mu_0 \ell I_1 I_2}{2\pi\delta}$
$\vec{\tau} = \vec{\mu}\times\vec{B} = (\mu B\sin\theta, \text{RHR})$

21 Electromagnetic Induction

$\mathcal{E} = \ell|\vec{v}||\vec{B}| = IR$
$\Phi_B = \vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ (uniform $\vec{B}$-field)
$\Phi_B = \int_{\text{loop area}} \vec{B}\cdot d\vec{A}$
$\mathcal{E} = \frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$
$\mathcal{E}_{\text{coil}} = N_t\left|\frac{\mathrm{d}\Phi_{B,\text{per coil}}}{\mathrm{d}t}\right|$
$\mathcal{E}_{\text{coil}} = \omega A|\vec{B}|N_t \sin\omega t$
$V_2 = \frac{N_{t2}}{N_{t1}}V_1$

22 Electromagnetic Fields and Waves

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$
$\oint \vec{B}\cdot d\vec{A} = 0$
$\oint \vec{E}\cdot d\vec{s} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} + \varepsilon_0\mu_0\frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$
$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$
$I_{\text{disp}} = \varepsilon_0\frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$
$\vec{v}_{\text{EM wave}} = c_0 = (\varepsilon_0\mu_0)^{-0.5}$
$c_0 = \lambda f$
$|\vec{E}| = c|\vec{B}|$
$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$
$E_e = \frac{P}{A} = \langle\vec{S}\rangle = \frac{1}{2c_0\mu_0}E_0^2 = \frac{c_0\varepsilon_0}{2}E_0^2$
$P_{\text{EM}} = \frac{F}{A} = \frac{E_e}{c_0}$ (perfect absorber)
$E_e = E_{e0}\cos^2\phi$
$E_{\text{e,transmitted}} = \frac{1}{2}E_{e0}$ (incident light unpolarised)

23 Physical Constants

$\varepsilon_0 = 8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$ or $\frac{\text{F}}{\text{m}}$
$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
$e = 1.6\times10^{-19}$ C
$m_e = 9.11\times10^{-31}$ kg
$m_p = 1.67\times10^{-27}$ kg
$\mu_0 = 1.26\times10^{-6}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$
$c_0 = 3.00\times10^8\ \frac{\text{m}}{\text{s}}$

24 Useful Geometry

Circle:
$A = \pi r^2$
$C = 2\pi r$
Sphere:
$A_s = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$
Cylinder
$A_s = 2\pi r\ell$ (+ ends if closed)
$V = \pi r^2 \ell$

25 Supplemental: Simple Harmonic Motion and Waves

$\omega = 2\pi f \qquad T = \frac{1}{f} \qquad \vec{v}_\phi = \lambda f$
$\omega_{\text{SHM}} = \sqrt{\frac{k}{m}} = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{mg\ell}{I}}$
$y(x,t) = A\sin(kx - \omega t + \phi) \qquad k \equiv \frac{2\pi}{\lambda}$
$E_k = 0.5kA^2$
$\vec{v}_{\text{string}} = \sqrt{\frac{\vec{F}_t}{\mu}} \qquad \mu \equiv \frac{m}{\ell}$
Open cylinder: $f_n = n\frac{\vec{v}}{2\ell} \quad n \in \mathbb{N}$
Closed cylinder: $f_n = n\frac{\vec{v}}{4\ell} \quad n \in [2\mathbb{N}^* + 1]$

26 Supplemental: Optics

$n = \frac{c_0}{\vec{v}_\phi} \qquad n_1\sin\theta_1 = n_2\sin\theta_2$
$\theta_c = \arcsin\frac{n_2}{n_1}\Big|_{n_1>n_2} \qquad \lambda = \frac{\lambda_0}{n}$
Thin lens: $\frac{1}{\ell_f} = \frac{1}{s_{\text{img}}} + \frac{1}{s_{\text{obj}}}$
$\ell_f = -\frac{R}{2}$
$M = \frac{h_{\text{img}}}{h_{\text{obj}}} = -\frac{s_{\text{img}}}{s_{\text{obj}}}$

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